Streamwood High School

AP Calculus AB

3.5-3.7, 3.9 EXAM REVIEW

Name Key

Limits at Infinity/Conclusions about *f(x)* given *f'(x)*/Optimization/Differentials

Find the horizontal asymptote(s), if any.

1.) $y=\dfrac{x^2-x-6}{x^2-4}$ $y=1$

2.) $y=\dfrac{4}{2x^2+7x-15}$ $y=0$

3.) $y=\dfrac{7x+1}{6x^2+7}$ $y=0$

4.) $y=\dfrac{-3x-1}{-5x-4}$ $y=\frac{3}{5}$

5.) $y=\dfrac{-5x^2-1}{3x}$ NONE

6.) $y=\dfrac{-5x}{\sqrt{64x^2-3}}$ # HAS 2 $y=-\frac{5}{8}$

use $y=\dfrac{-5x}{8|x|}$ $y=\frac{5}{8}$

7.) $f(x)=\dfrac{x}{1+x^4}$ $y=0$

8.) $f(x)=\dfrac{x^2}{x^2+1}$ $y=1$

9.) $f(x)=x\sqrt{9-x^2}$ None

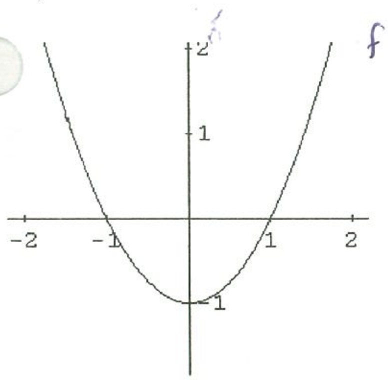
Domain of x is only $[-3,3]$

No big picture

For 10 & 11, the graph of the derivative of the function, f' is shown. Identify the intervals where *f(x)* is increasing and decreasing, local extrema, intervals where *f(x)* is concave up or down, and any points of inflection.

NOTE: Domain stops at ± 4

10) Domain $[-2,2]$



$f'(x)$

Increasing $[-2,-1)$ + $(1,2]$

Decreasing $(-1,1)$

Local Min $x=1$

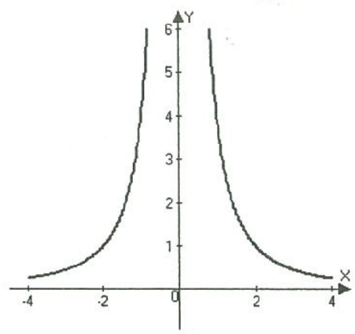
Local Max $x=-1$

Concave Up $(0,2)$

Concave Down $(-2,0)$

Point of Inflection $x=0$

11.)



Increasing $[-4,0)$ + $(0,4]$

Decreasing N/A

Local Min N/A

Local Max N/A

Concave Up $[-4,0)$

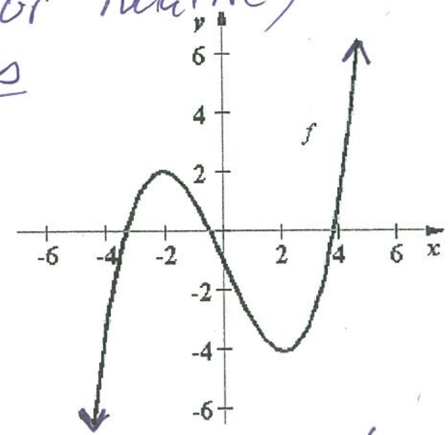
Concave Down $(0,4]$

Point of Inflection N/A

12a. If $f'(x)$ is decreasing then $f(x)$ is Concave down.

b. If $f'(a) = 0$ & $f'(x)$ changes signs from negative to positive at $x = a$, then $f(a)$ is a local Minimum (or relative).

The graph of f is shown to the right. use domain of all reals



13.) Name the x-intercept(s) of f'. $x = -2, 2$

14.) On what interval(s) is $f' > 0$?

$(-\infty, -2)$ & $(2, \infty)$

15.) For which value of x is f'' zero?

$x = 0$

16.) Find two positive numbers whose product is 187 and whose sum is a minimum.

$x \cdot y = 187$

$y = \frac{187}{x}$

Sub in $x + y = m$

$x + \frac{187}{x} = m$

$x + 187x^{-1} = m$

$m' = 1 - 187x^{-2}$

$0 = 1 - \frac{187}{x^2}$

$\frac{187}{x^2} = 1$

$x^2 = 187$

$x = \sqrt{187}$

$\sqrt{187}$ and $\sqrt{187}$

17.) Find the length and width of a rectangle that has an area of 288 square feet and whose perimeter is a minimum.

$P = 2L + 2w$

$288 = L \cdot w$

$\frac{288}{w} = L$

sub in

$P = \frac{576}{w} + 2w$

$P = 576w^{-1} + 2w$

$P' = -576w^{-2} + 2$

$0 = \frac{-576}{w^2} + 2$

$\frac{576}{w^2} = \frac{2}{1}$

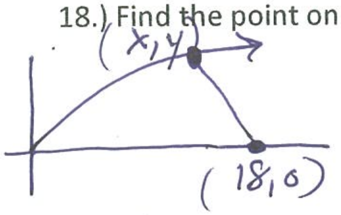
$\frac{2w^2}{2} = \frac{576}{2}$

$w^2 = 288$

$w = \sqrt{288}$

$L = \sqrt{288}$

18.) Find the point on the graph of the function $f(x) = \sqrt{x}$ that is closest to the point $(18, 0)$.



$y = \sqrt{x}$

$d = \sqrt{(x-18)^2 + y^2}$

$d = \left(x^2 - 36x + 324 + (\sqrt{x})^2\right)^{\frac{1}{2}}$

$d = \left(x^2 - 36x + 324 + x\right)^{\frac{1}{2}}$

$d = \left(x^2 - 35x + 324\right)^{\frac{1}{2}}$

$d' = \frac{1}{2}\left(x^2 - 35x + 324\right)^{-\frac{1}{2}}(2x - 35)$

$d' = \frac{2x - 35}{2\sqrt{x^2 - 35x + 324}}$

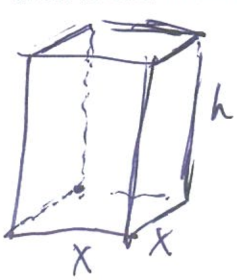
Let $d' = 0$ so solve

$2x - 35 = 0$

$x = 17.5$

The point is $(17.5, \sqrt{17.5})$.

19.) Determine the dimensions of a rectangular solid (with a square base) with maximum volume if its surface area is 121 meters^2.



$121 = 2x^2 + 4xh$

$\frac{121 - 2x^2}{4x} = \frac{4xh}{4x}$

$\frac{121 - 2x^2}{4x} = h$

Solve for h to sub in your primary eq

$V = x \cdot x \cdot h$

$V = x^2 h$

$V = x^2\left(\frac{121 - 2x^2}{4x}\right)$

So $V = \frac{121}{4}x - \frac{1}{2}x^3$

$V' = \frac{121}{4} - \frac{3}{2}x^2$ Set $V' = 0$

$0 = \frac{121}{4} - \frac{3}{2}x^2$

$\frac{3}{2}x^2 = \frac{121}{4}$

$x^2 = \frac{121}{4} \cdot \frac{2}{3}$

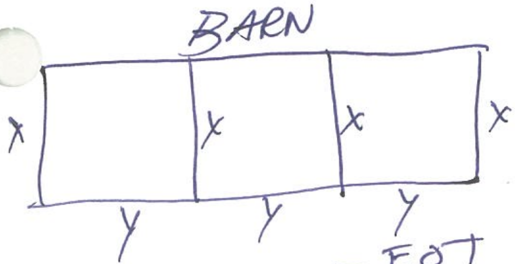
$x^2 = \frac{121}{6}$

$x = \sqrt{\frac{121}{6}} \approx 4.491\,m$

$h = $ ~~le m~~ $4.491\,m$ by $4.491\,m$ by $4.491\,m$

20.) A farmer needs to make 3 rectangular pig pens against a barn (one side is the barn). What is the maximum area a farmer can fence in with 400 feet of fencing?

$A = 3xy$ or $A = 3y \cdot x$



$4x + 3y = 400$

$3y = 400 - 4x$

$A = (400 - 4x)x$

$A = 400x - 4x^2$

$A' = 400 - 8x$

$0 = 400 - 8x$

$x = 50$

$3y = 400 - 200$

$3y = 200$

$A = 200(50)$

$\boxed{A = 10{,}000\ ft^2}$

EOT

21.) Find the linearization, *L(x)*, for $f(x) = \frac{23}{x^2}$ at x = 1.

$f(x) = 23x^{-2}$

Pt $(1, 23)$

$f'(x) = -46x^{-3}$

$f'(1) = -46$

$L(x) = 23 - 46(x - 1)$

22.) Determine the ***approximate change in y*** for $y = -2x^2 - 4$ at $x = 2$ with $\Delta x = dx = 0.05$. Give your answer to 3 decimal places.

$dy = -4x\,dx$

$dy = -4(2)(.05)$

$dy = -.400$

23.) Determine the ***approximate value*** at x = 0.1 for $y = x^4 - 4$ using differentials or linearization.

use $x = 0$ as the starting point $(0, -4)$

VALUE $= -4 + dy$
$-4 + 0$

$dy = 4x^3\,dx$

$dy = 4(0)^3(.1) = 0$

so the approx. value at $x = .1$ is ≈ -4

Find the differential dy **of the function.**

24.) $y = -x^2 + 3x + 4$

$dy = (-2x + 3)\,dx$

25.) $y = x\sin(5x)$

$dy = [5x\cos 5x + \sin 5x]\,dx$

26.) Given *f*(2) = 3 and *f '*(2) = ½, approximate the value of *f*(2.3).

use EOT

$y - 3 = \frac{1}{2}(x - 2)$

$y = 3 + \frac{1}{2}(.3)$

$y = 3 + .15$

$f(2.3) \approx 3.15$

27.) Given $f(x) = e^x$, use differentials or linearization to estimate *f*(-0.1).

use $x = 0$ Pt $(0, 1)$

$f'(x) = e^x$

$f'(0) = e^0 = 1$

$L(x) = 1 + 1(x - 0)$

$L(-.1) = 1 - .1$

$L(-.1) = .9$

so $f(-.1) \approx .9$

28.) Use differentials or linearization to approximate the value of √104 with a rational number.

$f(x) = \sqrt{x}$ start w/ $x = 100$ $dx = 4$ PT: $(100, 10)$

$dy = \frac{1}{2}x^{-\frac{1}{2}}\,dx$

$dy = \frac{1}{2} \cdot \frac{1}{\sqrt{100}} \cdot \frac{4}{1}$ ✓

$dy = \frac{4}{20}$

$dy = \frac{1}{5}$ or $.2$

VALUE $= 10 + dy$

so $\boxed{\sqrt{104} \approx 10.2}$